Electron and Quasiparticle Exponents of Haldane-Rezayi state in Non-abelian Fractional Quantum Hall Theory

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Abstract

The quasiparticle propagator of Haldane-Rezayi(HR) fractional quantum Hall (FQH) state is calculated, based on a chiral fermion model (or a Weyl fermion model) equipped with a hidden spin $SU(2)$ symmetry. The spectrum of the chiral fermion model for each *total spin* and total momentum is shown to be identical to that of the $SU(2)$ $c = -2$ model introduced to describe the edge spectrum of the HR state.

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I. INTRODUCTION

Incompressible quantum Hall liquids form a new class of matter states which contain non-trivial topological orders. It is known that quantum Hall states can be divided into two classes: abelian quantum Hall state and non-abelian quantum Hall state. [1] Almost all observed quantum Hall states are abelian states. Although the observed filling fraction $\nu = 5/2$ state may be a non-abelian states, we still have no direct evidence of non-abelian state in experiments. To confirm a non-abelian state in experiments, we need to use tunneling experiments to measure exponents of electron (or quasiparticle) propagator along the edge.

Many non-abelian incompressible FQH fluids have been the subject of active theoretical research for the last few years. The notion of non-abelian quantum Hall states was first proposed by Moore and Read [2], largely motivated by an observation that many-body FQH wave functions can be constructed as a correlation function in a 2d conformal field theory (CFT). Later non-abelian quantum Hall states were also constructed through parton construction that leads to effective theories with non-abelian gauge field. [3] However the two approaches were shown to be closely related to each other. [4] The non-abelian states are characterized by the existence of non-abelian quasiparticles. Although theoretically we still cannot prove the non-abelian Berry's phases induced by exchanging two quasiparticles, some evidences for their non-abelian statistics have recently been tested and confirmed through the counting of number of degeneracy of quasiparticle excitations. [5]

The non-abelian states can also be characterized (more completely) through their edge structure. This characterization allows us to identify non-abelian states in experiments. The edge excitations of any abelian states can be described by edge phonon theory with several branches (more precisely, by several Gaussian models or by several $U(1)$ current algebras). While the edge excitations of a non-abelian state is described by a theory which cannot be identified as the Gaussian models. The abelian and non-abelian states have different sets of exponents for electron and quasiparticle propagators.

The edge theory for a non-abelian state can be obtained through the parton construction

that produces the non-abelian bulk state. [3] It can also be obtained through a (conjectured) relation between the bulk CFT that produces the bulk wave function and the edge CFT that describes the edge excitations. [6] For all known abelian and non-abelian states, the edge CFT was always identical to the minimal bulk CFT. [7,8] It was shown in Ref. [7] (under certain assumptions) that the edge CFT (which is identified with the minimal bulk CFT) always reproduces the low energy spectrum of edge excitations. In many case (such as abelian states and the Pfaffian state), the natural inner product of the edge CFT is also identical to the inner product of edge excitations defined through the electron wave functions. (This is one of big unsolved mysteries, and there are exceptions.) For those quantum Hall states, we may use the known correlations in the edge CFT to calculate electron and quasiparticle propagators along the edge. Both edge spectra and the electron propagators obtained from edge CFT have been confirmed by direct numerical calculations for several non-abelian states.

One of the most important non-abelian states is the HR state for spin 1/2 electrons [9]. The HR state was proposed to explain the plateau at filling fraction $\nu = 5/2$ observed in experiment [10] and thus may serve as the first candidate of real non-abelian FQH states. There are two approaches for the HR state. One is based on the $SU(2)$ $c = -2$ CFT (plus an abelian U(1) Gaussian part). [7,8] Comparing to the minimal $c = -2$ CFT, the $SU(2)$ $c = -2$ CFT has an additional $SU(2)$ symmetry and primary fields of dimension $h_s = \frac{1}{8}[(4s+1)^2-1]$ which form an irreducible representation of $SU(2)$ with spin s. In the rest of the paper, we will simply refer the $SU(2)$ $c = -2$ CFT as the $c = -2$ CFT. The edge spectrum can then be analyzed by the known character of CFT in the $c = -2$ model. However, due to its nonunitarity, the $c = -2$ model has negative-norm states in its Hilbert space if one uses its natural inner product. Presumably, the inner product between physical edge states and that between the CFT states are not the same. Although this drawback does not ruin the CFT description of the spectrum, it does prohibit us from doing any further calculation, such as electron and quasiparticle propagators along the edge, based on the CFT techniques. The other approach [11] uses a free field realization of the $c = -2$ CFT, with a lagrangian

$L = |\partial_t \eta|^2 - |\partial_x \eta|^2$ where η is a complex fermionic field. The standard quantization leads to the $c = -2$ theory with negative-norm states. However its was shown in Ref. [11] that one can redefine the inner product which remove all negative-norm states and *leave the correlations between electron operators unchanged*. Thus the new theory with the redefined inner product can still reproduces the bulk wave function of HR state through correlations of electron operators. In this way, the exponent of electron propagator is found to be 4 which agrees with value obtained form CFT. [8]. The exponent of quasiparticle propagator is still unknown. If we had used CFT results, the exponent would be $-1/8$, which is physically unacceptable. In this paper, we will calculate the exponent of quasiparticle propagator and the exponent will be found to be $+3/8$. We will also present some numerical results that support the exponent 4 for the electron propagator.

The Vector space of the $c = -2$ CFT is identical to that of a chiral fermion theory (or, in string theory terminology, a Weyl fermion or two Majorana-Weyl fermions). [11,12] After the redefinition of the inner product for the $c = -2$ CFT, the two theories even have the same inner product. [11,12] The authors of Ref. [13] used the chiral fermion theory to describe the edge excitation of HR state and found the electron exponent to be 3. This result disagrees with the result from Ref. [11,8], and is not favored by our numerical result.

In section two we show that the low energy edge excitations of the HR state generated by the $c = -2$ CFT is equivalent to those generated by a chiral fermion (in the twisted sector). In section three we identify the electron operators constructed in [11] and calculate their correlation function, which reproduces the HR wave function. We then use these electron operators to define quasiparticle operators by imposing the appropriate boundary conditions. This allows us calculate the exponent of quasiparticle propagator. No CFT calculation was used. Finally we generalized this technique to other non-abelian FQH states. The quasiparticle exponents calculated based on this technique are consistent with known results for those non-abelian FQH states.

II. THE EDGE SPECTRUM

The HR state is a d-wave paired spin singlet FQH state with filling fraction $\nu = 1/m$:

$$\Phi_{HR}(z_i, w_i) = \Phi_m \Phi_{ds}(z_i, w_i),$$
$$\Phi_{ds} = \mathcal{A}_{z,w}(\frac{1}{(z_1 - w_1)^2}\frac{1}{(z_2 - w_2)^2}...),$$
$$\Phi_m = (\prod_{i<j}(z_i - z_j)^m \prod_{i<j}(w_i - w_j)^m \prod_{i<j}(z_i - w_j)^m) \exp(-\frac{1}{4}\sum_i(|z_i|^2 + |w_i|^2)), \quad (2.1)$$

where $\mathcal{A}_{z,w}$ is the anti-symmetrization operator which performs separate anti-symmetrizations among z_i's and among w_i's ; m is an even integer. Here $z_i(w_i)$ are the coordinates of the spin-up (-down) electrons. The edge spectrum of a circular droplet of HR state is labeled by total angular momentum and forms a representation of the spin $SU(2)$. Let $N^{nab}_{L,s}$ be the number of edge excitations for the non-abelian part (we will ignore the abelian U(1) part) which carry the following quantum numbers: the total angular momentum is L, the total spin is s and the z-component of the total spin is $S_z = 0$ (or 1/2 if s is a half-odd-integer). $N^{nab}_{L,s}$ was described in Ref. [8] by the following character in the $c = -2$ model

$$Ch_s(\xi) \equiv \sum_{n=0}^{\infty} N^{nab}_{L_0+n,s}\xi^n = \frac{\xi^{h_s} - \xi^{h_{s+1/2}}}{\prod_{n=1}^{\infty}(1 - \xi^n)} \quad (2.2)$$

where $h_s = \frac{1}{8}[(4s+1)^2 - 1]$ and L_0 is the total angular momentum of the ground state in the spin $s = 0$ sector. Multiplying Eq. (2.2) by $(2s + 1)$ gives us number of edge states in each total spin s sector. On the other hand, we can also use the following Hamiltonian proposed in Ref. [11]

$$H = v\sum_{k=1}^{\infty} k(c^\dagger_{k\uparrow}c_{k\uparrow} + c^\dagger_{k\downarrow}c_{k\downarrow}),$$
$$\{c^\dagger_{k\sigma}, c_{k'\sigma'}\} = \delta_{kk'}\delta_{\sigma\sigma'},$$
$$\{c_{k\sigma}, c_{k'\sigma'}\} = \{c^\dagger_{k\sigma}, c^\dagger_{k'\sigma'}\} = 0 \quad (2.3)$$

to describe the edge excitations of HR state. Here v is the edge velocity (which will be set to $v = 1$ in the following discussion) and we have put the system on a circle $x \in [0, 2\pi)$. The Hamiltonian in Eq. (2.3) has a global $SU(2)$ symmetry with the total spin generators [11]

$$
\begin{aligned}
S^z &= \frac{1}{2}\sum_{k=1}^{\infty}(c^\dagger_{k\uparrow}c_{k\uparrow} - c^\dagger_{k\downarrow}c_{k\downarrow}),\\
S^+ &= \sum_{k=1}^{\infty} c^\dagger_{k\uparrow}c_{k\downarrow},\\
S^- &= \sum_{k=1}^{\infty} c^\dagger_{k\downarrow}c_{k\uparrow}.
\end{aligned}
\tag{2.4}
$$

We have tabulated the spectrum of Eq. (2.3) for up to 6th low-lying energy eigenstates for each spin sector with $s \le 3/2$. All results are the same as those generated by the character in Eq. (2.2). To prove a general relation, let $N_{K,s}$ be number of states of Eq. (2.3) with total momentum K, total spin s and total z-component of spin $S_z = 0$. Let $n_{K,M}$ be number of states with total momentum K and total z-component of spin $S_z = M$. It is clear that $N_{K,s} = n_{K,s} - n_{K,s+1}$. The character for $n_{K,M}$ can be calculated:

$$
\begin{aligned}
ch(\eta,\xi) &\equiv \sum_{K,M} n_{K,M}\eta^{2M}\xi^K\\
&= \prod_{k=1}^{\infty}(1+\eta\xi^k)(1+\eta^{-1}\xi^k)
\end{aligned}
\tag{2.5}
$$

To evaluate the product in Eq. (2.5), we consider the follow (twisted) chiral fermion model

$$
H = \sum_{k=0,\pm1,\pm2,\ldots} k : c^\dagger_k c_k := \sum_{k=1,2,\ldots} k c^\dagger_k c_k + \sum_{k=-1,-2,\ldots} (-k) c_k c^\dagger_k \tag{2.6}
$$

We note that the model described by Eq. (2.6) is identical to that described by Eq. (2.3) except the $k = 0$ mode. Let $n^c_{K,Q}$ be number of states with total momentum K and total fermion number Q, where Q is given by

$$
Q = \sum_{k=1}^{\infty} c^\dagger_k c_k - \sum_{k=0}^{-\infty} c_k c^\dagger_k
$$

Note that an empty $k = 0$ level is regarded as the presence of a hole which contribute -1 to the total fermion number Q. Through bosonization we know that $n^c_{K,Q}$ is equal to the partition number p_{K-K_0} where $K_0(Q) = (Q+1/2)^2/2 - 1/8$ is the minimum momentum of states with Q fermions. Using $\sum_{i=0}^{\infty} p_i\xi^i = 1/\prod_{n=1}^{\infty}(1-\xi^n)$, we find

$$
ch^c(\eta,\xi) \equiv \sum_{K,Q} n^c_{K,Q}\eta^Q\xi^K
$$

$$= (1+\eta^{-1}) \prod_{k=1}^{\infty} (1+\eta\xi^k)(1+\eta^{-1}\xi^k) \tag{2.7}$$

$$= \sum_Q \frac{\xi^{K_0(Q)}\eta^Q}{\prod_{n=1}^{\infty}(1-\xi^n)} \tag{2.8}$$

The $(1+\eta^{-1})$ term is the character for the $k=0$ level. Compare Eq. (2.5) with Eq. (2.8), we see that

$$\begin{aligned} ch(\eta,\xi) &= ch^c(\eta,\xi)/(1+\eta^{-1}) \\ &= \sum_Q \frac{\eta^Q \sum_{n=0}^{\infty}(-)^n \xi^{K_0(Q+n)}}{\prod_{n=1}^{\infty}(1-\xi^n)} \end{aligned} \tag{2.9}$$

After replacing Q by $2M$ and noticing that $K_0(2M) = h_M$, we find

$$\begin{aligned} ch_M(\xi) &\equiv \sum_K n_{K,M}\xi^K \\ &= \frac{\sum_{n=0}^{\infty}(-)^n \xi^{h_{M+\frac{n}{2}}}}{\prod_{n=1}^{\infty}(1-\xi^n)} \end{aligned} \tag{2.10}$$

From the relation $N_{K,s} = n_{K,s} - n_{K,s+1}$ we find

$$Ch_s(\xi) = ch_s(\xi) - ch_{s+1}(\xi) = \frac{\xi^{h_s} - \xi^{h_{s+1/2}}}{\prod_{n=1}^{\infty}(1-\xi^n)} \tag{2.11}$$

which is exactly the character obtained from the $c = -2$ CFT. Thus the spectrum of Eq. (2.3) labeled by (angular) momentum and the total spin is identical to that of $c = -2$ CFT labeled by the same quantum number.

III. THE ELECTRON AND QUASIPARTICLE EXPONENTS

Finding the theory with positive definite inner product that completely describes the spectrum of edge excitations is not the end of story. We also need to identify the electron operators in the theory. One naive guess would be $\psi_\sigma e^{i\sqrt{m}\phi}$, where

$$\begin{aligned} \psi_\uparrow(t,x) &= \sum_{k=1}^{\infty} c_{k,\uparrow}e^{-ik(t-x)} + \sum_{k=1}^{\infty} c^\dagger_{k,\downarrow}e^{ik(t-x)}, \\ \psi_\downarrow(t,x) &= \sum_{k=1}^{\infty} c_{k,\downarrow}e^{-ik(t-x)} - \sum_{k=1}^{\infty} c^\dagger_{k,\uparrow}e^{ik(t-x)} \end{aligned} \tag{3.1}$$

This is the choice made in Ref. [13]. The correlation of $\psi_\sigma e^{i\sqrt{m}\phi}$ contains two parts. The first part comes from $e^{i\sqrt{m}\phi}$ which reproduces the abelian part $\prod(z_i-z_j)^m(z_i-w_j)^m(w_i-w_j)^m$ of the HR wave function. But the second part coming from ψ_σ fails to reproduce the non-abelian part $\mathcal{A}_{z,w}(\frac{1}{(z_1-w_1)^2}\frac{1}{(z_2-w_2)^2}...)$ of the HR wave function. In this section we will follow the following principle to choose the electron operators. We will require the electron operators to reproduce the bulk wave function. Such a principle, although has not been proven to be correct, has led to correct electron operators for abelian states and for a non-abelian Pfaffian state. This principle leads us to the electron operator introduced in Ref. [11]:

$$\psi_{e,\sigma}e^{i\sqrt{m}\phi} \tag{3.2}$$

where

$$\begin{aligned}
\psi_{e\uparrow}(t,x) &= \sum_{k=1}^{\infty}\sqrt{k}c_{k,\uparrow}e^{-ik(t-x)}+\sum_{k=1}^{\infty}\sqrt{k}c^\dagger_{k,\downarrow}e^{ik(t-x)},\\
\psi_{e\downarrow}(t,x) &= \sum_{k=1}^{\infty}\sqrt{k}c_{k,\downarrow}e^{-ik(t-x)}-\sum_{k=1}^{\infty}\sqrt{k}c^\dagger_{k,\uparrow}e^{ik(t-x)}
\end{aligned} \tag{3.3}$$

The correlator of $\psi_{e,\sigma}$ can then be calculated to be

$$<\psi_{e\uparrow}(z_1)\psi_{e\downarrow}(w_1)\psi_{e\uparrow}(z_1)\psi_{e\downarrow}(w_1)...>=\mathcal{A}_{z,w}(\frac{1}{(z_1-w_1)^2}\frac{1}{(z_2-w_2)^2}...), \tag{3.4}$$

which reproduces the non-abelian part of the HR wave function. We will take Eq. (3.2) as the electron operators near the edge. The electron propagator on the edge thus has the following form

$$G_e(t,x)\sim\frac{1}{(x-vt)^{g_e}},\qquad g_e=m+2. \tag{3.5}$$

When $m=2$, such a propagator with exponent $g_e=4$ leads to a set of occupation numbers that satisfies

$$n_{l_0}:n_{l_0-1}:...=1:4:10:20:35:... \tag{3.6}$$

Here n_l is the occupation number on the orbit with angular momentum l of a circular droplet, and l_0 is the last orbit occupied by electrons (*ie* $n_l=0$ for $l>l_0$). Direct numerical

calculation for three spin down and three spin up electrons gives $1 : 4.351 : 10.71 : 15.99 : ...$ and for four spin down and four spin up electrons gives $1 : 4.299 : 10.39 : 18.52 : 24.68 : ....$ If $g_e = 3$ we would have $1 : 3 : 6 : 10 : ...$, and if $g_e = 5$ we would have $1 : 5 : 15 : 35 : ....$ We see that numerical results almost rule out the possibility of $g_e = 3$ and $g_e = 5$, and give strong support for $g_e = 4$

We now turn to consider the quasiparticle propagator of HR state. The quasiparticle operator has a form $\eta_q e^{i\phi/2\sqrt{m}}$. [8] The non-abelian part η_q creates a cut of -1 for the operator $\psi_{e,\sigma}$, *ie* $\psi_{e,\sigma}$ changes sign as it goes around η_q. Note that the abelian part $e^{i\phi/2\sqrt{m}}$ also create a cut of -1 to $e^{i\phi\sqrt{m}}$, the abelian part in the electron operator. Thus the total quasiparticle operator $\eta_q e^{i\phi/2\sqrt{m}}$ does not generate any cut to the total electron operator $\psi_{e,\sigma} e^{i\phi\sqrt{m}}$, as required by the single-valueness of the electron wave function. It was also shown in Ref. [8] that the quasiparticle carries $1/2m$ charges and zero spin. In the follow we will concentrate on non-abelian part of quasiparticle operator. First we write down quasiparticle operators using the prescribed twisted boundary condition between the non-abelian parts of electron and quasiparticle operators. Introduce $U(x_1, x_2)$ be the product of two η_q fields at equal time: $U(x_1, x_2) = \eta_q(x_1)\eta_q(x_2)$. Such an operator $U(x_1, x_2)$ can be defined through its commutator with $\psi_{e,\sigma}$

$$U(x_1, x_2)\psi_{e,\sigma}(x) = f(x)\psi_{e,\sigma}(x)U(x_1, x_2) \tag{3.7}$$

where $f(x)$ is defined to be

$$f(x) = \begin{cases} -1, & x_1 < x < x_2 \\ +1, & \text{otherwise} \end{cases} \tag{3.8}$$

The quasiparticle propagator (at equal time) is then given by

$$< 0|U(x_1, x_2)|0 > . \tag{3.9}$$

As an example, we first work on the case of a chiral fermion model

$$\psi(t, x) = \sum_{k \in Z} c_k e^{-ik(t-x)}, \tag{3.10}$$

which corresponds to two-critical-Ising model. Since $\psi(x)$ has the standard mode expansion, we will do it on the coordinate space. To solve Eq. (3.7), we first write it in the following form

$$U^{\alpha}_{2I}(x_1,x_2)\psi(x)U^{\alpha,-1}_{2I}(x_1,x_2) = e^{i\frac{\pi}{2}\alpha(f(x)-1)}\psi(x) \tag{3.11}$$

where α is a real parameter. For $\alpha = 1$ one gets Eq. (3.7). Since

$$[\psi(x), \frac{1}{4}\int dy\psi^{\dagger}(y)\psi(y)(f(y)-1)] = \frac{\pi}{2}(f(x)-1)\psi(x) \tag{3.12}$$

we find

$$U_{2I}(x_1,x_2) = \exp\left(\frac{i}{4}\int dx\psi^{\dagger}(x)\psi(x)(f(x)-1)\right). \tag{3.13}$$

The quasiparticle propagator can now be easily calculated by using bosonization rule by noting $\psi^{\dagger}(x)\psi(x)$ is 2π times the fermion density: $\psi^{\dagger}(x)\psi(x) = 2\pi\rho = \partial\phi$. We find $U_{2I}(x_1,x_2) = e^{i(\phi(x_1)-\phi(x_2))/2}$ and

$$< 0|U_{2I}(x_1,x_2)|0 >= (x_1-x_2)^{-1/4}. \tag{3.14}$$

The exponent in Eq. (3.14) is consistent with the result of two-critical-Ising model by using the standard CFT calculation.

We now apply Eq. (3.7) to the case of operators $\psi_{e,\sigma}$. We will use a different approach to calculate the average $< 0|U(x_1,x_2)|0 >$. First we note that $\psi_{e\uparrow}$ can be written as

$$\psi_{e\uparrow} = \sum_{k=1}^{\infty}\sqrt{k}c_{k,\uparrow}e^{-ik(t-x)} + \sum_{k=-1}^{-\infty}\sqrt{-k}c_{k,\uparrow}e^{-ik(t-x)} \tag{3.15}$$

if we rename $c^{\dagger}_{k,\downarrow}$ by $c_{-k,\uparrow}$ for $k>0$. The ground state $|0>$ can be regarded as a filled Fermi sea with negative momentum states occupied by spin $\uparrow$ electron, *ie* $c^{\dagger}_{k,\uparrow}|0>|_{k<0} = 0$. Using the first-quantized picture, we write

$$< 0|U(x_1,x_2)|0 >= \lim_{M\to\infty} < null|\left(\prod_{m=1}^{M} c_{-m,\uparrow}\right)U(x_1,x_2)\left(\prod_{m=1}^{M} c^{\dagger}_{-m,\uparrow}\right)|null > \tag{3.16}$$

where $|null>$ is the first-quantized vacuum with no fermions, and M is a momentum cutoff. According to Eq. (3.7), we know the operation of $U(x_1,x_2)$ on $c^{\dagger}_{-n,\uparrow} = c_{n,\downarrow}$ to be

$$\begin{aligned}
&U(x_1,x_2)c^\dagger_{-n,\uparrow} = U(x_1,x_2)c_{n,\downarrow} \\
&= \sum_{m>0}\sqrt{\frac{m}{n}} f_{nm} c_{m,\downarrow} U(x_1,x_2) - \sum_{m<0}\sqrt{\frac{m}{n}} f_{nm} c^\dagger_{-m,\uparrow} U(x_1,x_2) \\
&= \sum_{m>0}\sqrt{\frac{m}{n}} f_{nm} c^\dagger_{-m,\uparrow} U(x_1,x_2) - \sum_{m<0}\sqrt{\frac{m}{n}} f_{nm} c^\dagger_{-m,\uparrow} U(x_1,x_2);
\end{aligned} \tag{3.17}$$

whereas the operation of $U_{2I}(x_1,x_2)$ on $c^\dagger_{-m}$ is

$$U_{2I}(x_1,x_2)c^\dagger_{-n} = \sum_m f_{nm} c^\dagger_{-m} U_{2I}(x_1,x_2), \tag{3.18}$$

where $f_{nm}(x_1,x_2)$ is defined to be

$$f_{nm} = \frac{1}{2\pi}\int dx e^{i(n-m)x} f(x). \tag{3.19}$$

Eq. (3.16) can then be calculated by using Eq. (3.17):

$$< 0|U(x_1,x_2)|0 >= \det(\sqrt{\frac{m}{n}} f_{nm}) = \det(f_{nm}) =< 0|U_{2I}(x_1,x_2)|0 > \tag{3.20}$$

where $(\sqrt{\frac{m}{n}} f_{nm})$ and (f_{nm}) are $M\times M$ matrices whose matrix elements are given by $\sqrt{\frac{m}{n}} f_{nm}$ and $f_{nm},\ m,n = 1,...,M$. In Eq. (3.20) we have used the fact that $\det(f_{nm})$ is just $< 0|U_{2I}(x_1,x_2)|0 >$ expressed in the first-quantized picture. We also have used

$$< null|U(x_1,x_2)|null >= \text{const}. \tag{3.21}$$

This is because $|null>$ is a state with no fermions and hence $U(x_1,x_2)$ cannot affect $|null>$. The exponent calculated this way is thus the same as that of Eq. (3.14). The total quasiparticle propagator is thus

$$G_q(t,x) \sim \frac{1}{(x-vt)^{g_q}}, \qquad g_q = \frac{1}{4} + \frac{1}{4m}. \tag{3.22}$$

where $\frac{1}{4m}$ is the contribution from the abelian part of the quasiparticle operator. The exponents g_e in Eq. (3.5) and g_q in Eq. (3.22) can be measured in tunneling experiment.

IV. QUASIPARTICLE EXPONENTS OF OTHER FQH STATES

The technique we have used to calculate the quasiparticle exponent can be generalized to many other non-abelian FQH states. First let us consider the generalized HR state given by

$$
\begin{aligned}
\Phi_{HR}(z_i, w_i) &= \Phi_m \Phi_{ds}(z_i, w_i), \\
\Phi_{ds} &= \mathcal{A}_{z,w}\Big(\frac{1}{(z_1-w_1)^{2l+2}}\frac{1}{(z_2-w_2)^{2l+2}}...\Big), \\
\Phi_m &= \Big(\prod_{i<j}(z_i-z_j)^m \prod_{i<j}(w_i-w_j)^m \prod_{i<j}(z_i-w_j)^m\Big)\exp\Big(-\frac{1}{4}\sum_i(|z_i|^2+|w_i|^2)\Big),
\end{aligned}
\tag{4.1}
$$

This wave function can be generated by an electron operator whose non-abelian part is $\partial^l \psi_{e,\sigma}$. We would like to point out that only for $l=0$ can $\mathcal{A}_{z,w}(\frac{1}{(z_1-w_1)^{2l+2}}\frac{1}{(z_2-w_2)^{2l+2}}...)$ be a correlation of a primary field of a CFT. For other l, it is a correlation of descendant field. The operator that produces a cut of -1 to $\psi_{e,\sigma}$ also produces a cut of -1 to $\partial^l \psi_{e,\sigma}$. Thus, for generalized HR state, although the electron exponent is changed to $g_e = m + 2l + 2$, the quasiparticle exponent remains the same $g_q = \frac{1}{4} + \frac{1}{4m}$. If we calculate the quasiparticle correlation using the first-quantized picture, we find the non-abelian part has a correlation $< 0|U(x_1, x_2)|0 >= \det(\left(\frac{m}{n}\right)^{l+\frac{1}{2}} f_{nm})$ which is independent of l as expected. This provides an important consistency check of the formalism we introduce in this paper. We will see next that the similar behavior is also shared by several other non-abelian QH states.

Now let us consider two other types of non-abelian states. The first one is the generalized Pfaffian wave function [2]

$$
\begin{aligned}
\Phi_p &= \Phi_{Pf}\Phi_m, \\
\Phi_{Pf} &= \mathcal{A}\frac{1}{(z_1-z_2)^{2l+1}}\frac{1}{(z_3-z_4)^{2l+1}}..., \\
\Phi_m &= \Big(\prod_{i<j}(z_i-z_j)^m\Big)\exp\Big(-\frac{1}{4}\sum_i |z_i|^2\Big),
\end{aligned}
\tag{4.2}
$$

where $\mathcal{A}$ is the anti-symmetrization operator; l is an integer and m is an even integer. The second one is the generalized d-wave paired FQH states for spinless electrons [8]

$$
\begin{aligned}
\Phi &= \Phi_d \Phi_m, \\
\Phi_d &= \mathcal{S}\frac{1}{(z_1-z_2)^{2l+2}}\frac{1}{(z_3-z_4)^{2l+2}}..., \\
\Phi_m &= \Big(\prod_{i<j}(z_i-z_j)^m\Big)\exp\Big(-\frac{1}{4}\sum_i |z_i|^2\Big),
\end{aligned}
\tag{4.3}
$$

where $\mathcal{S}$ is the symmetrization operator; l is an integer and m is an odd integer. In the CFT approach, the non-abelian parts of electron operators for states (4.2) and (4.3) are identified

as $\partial^l\psi$, where ψ is a dimension 1/2 primary field in a $c = 1/2$ CFT for state (4.2) and is a dimension 1 primary field in a $c = 1$ CFT for state (4.3). For the case of $l = 0$ [7], the quasiparticle operator η satisfies the following operator product expansion (OPE)

$$\psi(z)\eta(0) \sim z^{-1/2}[\mu(0) + O(z)], \tag{4.4}$$

where μ is another operator. For the case of general l, we take l-times derivatives on Eq. (4.4). It is easy to see that the cut and the dimension of the quasiparticle operator η and hence the exponent g_q will not be affected by the differentiation. Both states have charge $1/2m$ quasiparticles. The electron and quasiparticle exponents are $(g_e, g_q) = (m+2l+1, \frac{1}{8}+\frac{1}{4m})$ for state (4.2), and $(g_e, g_q) = (m + 2l + 2, \frac{1}{8} + \frac{1}{4m})$ for state (4.3). [8]

The third example is the generalized d-wave-paired-spin-triplet FQH state for spin-1/2 electrons [8]

$$\begin{aligned}
\Phi_{DL}(z_i, w_i) &= \Phi_{kmn}\Phi_{dt}(z_i, w_i), \\
\Phi_{dt} &= \mathcal{S}_{z,w}\frac{1}{(z_1 - w_1)^{2l+2}}\frac{1}{(z_2 - w_2)^{2l+2}}\cdots, \\
\Phi_m &= \prod_{i<j}(z_i - z_j)^m \prod_{i<j}(w_i - w_j)^m \prod_{i<j}(z_i - w_j)^m \exp(-\frac{1}{4}\sum_i(|z_i|^2 + |w_i|^2)),
\end{aligned} \tag{4.5}$$

where $\mathcal{S}_{z,w}$ is the symmetrization operator which performs separate symmetrizations among z_i's and among w_i's; l is an integer and m is an odd integer. For the case of $l = 0$ [8, 7], the quasiparticle operator η satisfies the following operator product expansion (OPE)

$$\psi_\pm(z)\eta(0) \sim z^{-1/2}[\mu_\pm(0) + O(z)], \tag{4.6}$$

where the non-abelian part of the electron operators $\psi_\pm$ were identified as the currents of a $U(1) \times U(1)$ Gaussian model which is a $c = 2$ CFT, and $\mu_\pm$ are some other operators. For the case of general l, we take l-times derivatives on Eq. (4.6) as we did before. The charge $1/2m$ quasiparticle has an exponent $g_q = \frac{1}{4}+\frac{1}{4m}$. [8] The electron exponent is $g_e = m+2l+2$.

V. SUMMARY

In this paper we calculated the exponent $g_q = \frac{1}{4} + \frac{1}{4m}$ of the quasiparticle propagator $1/(x - vt)^{g_q}$ for the HR state with filling factor $\nu = 1/m$. Our calculation is based on two assumptions:

- The edge spectrum (the non-abelian part) is described by character Eq. (2.2). This is checked and confirmed (to certain low levels) by numerical calculations.
- The non-abelian part of electron operator has a propagator $< 0|\psi_{e,\sigma}\psi_{e,\sigma'}|0 >= \delta_{\sigma\sigma'}/(x - vt)^2$. This assumption is checked by the numerical results presented in this paper.

The two assumptions allow us to construct the Hamiltonian and commutators in Eq. (2.3).

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